Understanding Multiplication and Division using Word Problems

The discussion on these pages is helpful for working with students just beginning to understand the concepts of multiplication and division, as well as for students working on solving word problems. **A fundamental understanding of the connectedness of the four operations what they do and how they work is essential to being able to solve word problems as well as perform computations.**

- Begin with simple made up *word problems* (not equations with arbitrary-looking symbols) and then let the student solve them with his own methods.
- Allow the student to use a large set of counters that can be physically manipulated. Encourage him to solve the problems using the counters and/or to draw a picture (*model*). Lead him toward appropriate models if necessary, but realize that students can often get it on their own with enough opportunity and patience. Once a problem has been solved, by whatever method, ask/help the student to write an appropriate equation(s).
- Realize that there is often more than one way to solve a problem.
- Help the student make connections between the operations. It's a good idea, once a problem is solved, to have the child write other equations that are appropriate for the model.
- Help the student make connections between word problems, models and equations. You want her to be able to draw a model and write an equation for each word problem you give her. Likewise, given an equation, she should be able to make up a word problem and draw a model, and given a model (picture), she should be able to make up a word problem and write an appropriate equation.
- Have the student think about her answer. Does it make sense?

Multiplication and division problems are fundamentally different from (and often harder for students than) addition and subtraction problems. Consider these two problems:

1. Bob has 4 apples. Sue gave him 3 more apples. How many apples does Bob have now?
2. Bob has 4 bags of apples with 3 apples in each bag. How many apples does Bob have?

Notice the problems are similar in that they involve the same numbers and ask essentially the same question. The difference is in what the numbers mean in each problem. In #1, both numbers and the answer represent a certain number of apples. In #2, the 3 tells the number of apples in a bag, but the 4 tells how many bags (groups). There is more to think about in this problem! Another difference is this: addition and subtraction problems almost always involve counting by ones—something easy and familiar for most students. Multiplication and division involve the much more difficult skill counting by groups (skip-counting).

Pay attention to the language you use. Keep it simple at first.

Perhaps the student understands "give each child 4 apples," but not "give 4 apples per child".

Encourage the student to use the phrase "groups of" to indicate creating a number of equal groups: 3 groups of 5 is 15; three 5's is 15; and then 3 times 5 is 15 and the symbols.

Help the student understand the meaning of each quantity in the multiplication problem: 3 x 2 is 3 groups of 2 or 2 + 2 + 2. 2 x 3 is 2 groups of 3 or 3 + 3. Sure, 3 x 2 and 2 x 3 are the same quantity, but a student may not intuitively see that 3 groups of 2 is the same as 2 groups of 3 without seeing a model (a rectangular array is great here; just turn it and 3 rows of 2 becomes 2 rows of 3). Don't just state this as a rule; instead let the student have experience building the arrays and solving problems of both types.

Use the terms "shared by" and "equal groups" for division problems, rather than starting out with "divided by".

Multiplication and division are connected.
Multiplication involves counting groups of like size and determining how many are in all. The numbers being multiplied are called the factors. The result is called the product. Division names a missing factor in terms of the known factor and the product. Be sure to use both notations for division: $8 \div 4$ and $4\overline{)8}$. The answer to a division problem is called the quotient.

Following are descriptions and examples of the basic structure of these word problems for you to use in making up word problems for the student. Don't stick to the easiest ones; that is a disservice to the student. Embellish them with the student's name or things he likes to do. Add extraneous information. Let the student make up his own problems. Definitely mix them up! Remember to have (help) the student write an appropriate equation after she solves each problem. *If she uses division, writing a related multiplication equation is an important way to relate division back to multiplication. In the early stages of learning multiplication, writing a related addition equation is also important.*

- *Equal group problems with the whole unknown:*
  1. Bob has 4 bags of apples. There are 6 apples in each bag. How many apples does Bob have altogether?
  2. If apples cost 7 cents each, how much did Bob spend for 5 apples?
  3. Sue walked for 3 hours at 4 miles per hour. How far did she walk?

  #1 is an example of a repeated addition problem: 4 x 6 = 6 + 6 + 6 + 6 = 24. #2 and #3 are rate problems, but notice they also involve repeated addition. A good way to use models here is to have the student make 4 groups of 6 counters for #1, 7 counters (or pennies) for each of the 5 apples, or 4 counters for the first hour, 4 for the second, and 4 for the third. In the beginning, it is only important that the groups are separate and of equal size. As the student progresses, you can encourage him to arrange the counters in rectangular arrays, 4 rows of 6, for example.

- *Equal group problems with the size of the groups unknown:* The whole is known and must be partitioned into a known number of groups. Can be thought of as fair-sharing.

  1. Bob has 24 apples. He wants to share them equally among his 4 friends. How many apples will each friend get?
  2. Bob paid 35 cents for 5 apples. What was the cost of 1 apple?
  3. Sue walked 12 miles in 3 hours. How many miles did she walk per hour?

  In the beginning, the student may physically need to distribute the whole, one at a time, to the groups. That's ok. He'll develop faster skills and shortcuts with practice.

- *Equal group problems with the number of groups unknown:* The whole is known and must be measured off into groups of a known size. Can be thought of as repeated subtraction.

  1. Bob has 24 apples. He put them into bags containing 6 apples each. How many bags did he use?
  2. Jill bought some apples at 7 cents a piece. The total cost of her apples was 35 cents. How many apples did she buy?
  3. Sue walked 12 miles at a rate of 4 miles per hour. How many hours did this take Sue?

This is modeled by starting with the whole and repeatedly removing groups of the given size. Make sure the groups are kept separate from one another so that the number of groups can be counted. The idea of a rectangular array can be used once the student is comfortable with the process. You are given the size of a group, which is the length of a row. Arrange the total number of counters (the whole) into a rectangular array with rows of the proper length. The number of rows is the number of groups.

- *Area and rectangular arrays:* In a rectangle the product of the length and the width gives the area of the rectangle in square units. If the area and one dimension are known, division is used to find the missing dimension. Rectangular arrays of square counters are the best model. Be careful with units: 3 feet x 2 feet gives an area of 6 square feet. You can also do rectangular arrays of, for example, trees: 2 rows of 3 trees each is 6 total trees.

- *Multiplicative comparison problems:* Two different sets are being compared; one of the sets is some multiple of the other
  1. Bob picked 6 apples. Sue picked 4 times as many apples as Bob. How many apples did Sue pick?
  2. Sue picked 24 apples. She picked 4 times as many apples as Bob. How many apples did Bob pick?
  3. Sue picked 24 apples. Bob only picked 6 apples. How many times more apples did Sue *pick than Bob?*

  In #1, the visual model is that Sue picked 4 sets of 6 apples (as many as Bob each of 4 times). In #2, the whole is known and the number of groups is known (4), but we don't know how many in each group. We need fair-sharing to determine the size of each of the 4 groups. . In #3, the whole is again known, but now must be separated into an unknown # of groups, each of size 6. We can use repeated subtraction.

- *Combination problems:* All of these can be modeled by physically showing all possibilities, perhaps using lines to connect each item in one set to each item in the other set.

  1. Sue has 3 pairs of pants and 4 shirts that can all be worn together. How many different outfits consisting of a pair of pants and a shirt does she have?
  2. An experiment involves tossing a coin and rolling a die. How many different outcomes are possible?
  3. Sue bought some new pants and shirts that can all be worn together. He has a total of 12 different outfits. If he bought 3 pairs of pants, how many shirts did he buy?

Additional ideas for modeling multiplication and division:

- Don't be afraid to use 0's and 1's in your problems, but don't state arbitrary rules, either. Use counters to model what happens. What does 0 groups of 4 (or 4 groups of 0) look like? What about 1 group of 4 (or 4 groups of 1)?

- Division by 0? Of course, it's not allowed, but many children are told this without any reason given for it. Try having the child take 10 counters and divide them into 0 equal groups. Or take 10 counters and ask how many sets of 0 counters can be made.
- Include some division problems with remainders. Unlike those in most grade-school textbooks, real-life division problems don't usually come out nicely. What to do with the leftovers is worth a thorough discussion! Here are some possibilities:

  Without the context of a word problem, the remainder is simply the part left over (early grades understanding).

  If you are sharing 5 cookies among 2 people, how many will each person get?
  2, but what about the other cookie? Do you throw it away or divide it into smaller parts?

  A rope is 25 feet long. How many 7-foot jump ropes can be made?
  3 and you discard the leftover rope.

  The ferry can hold 8 cars. How many trips will it have to make to carry 25 cars across the river?
  4 trips—you must go up to next whole number to get all the cars across.

  9 children must be taken in two vans to a party. How many should ride in each van?
  You can't split a child in half, so you'll have to put 4 in one van and 5 in the other.

The next page contains more examples of multiplication and division word problems suitable for use with a student who needs help understanding the concepts of multiplication and division, as well as for students who need help reading a word problem and deciding what to do. Some embellishing ideas are also given; just be creative! These problems can easily be changed to suit the student: change the names, use larger numbers, add extraneous information, etc, to make them harder. *Remember, however, that a student struggling with word problems will benefit from being allowed to use physical models, smaller numbers, and simple words in the beginning.*

Multiplication and Division Word Problems

Straightforward:

1. There are 6 children in the room. Each child has 2 shoes. How many shoes are there?

2. Mary has 15 cookies. She gave 3 friends the same number of cookies each. How many cookies did each friend get?

3. Joe has 12 balloons. He gave each child 3 balloons. How many children were there?

4. Each balloon costs 10 cents. If Bill spent 40 cents on balloons, how many did he buy?

5. Stephanie wants to plant 8 apple trees behind her house in two equal rows. How many apple trees will be in each row?

6. Each hamburger costs $3 at the concession stand. If Stacy buys 4 hamburgers, how much will she spend?

7. Bill has 3 cookies. Laura has 3 times as many cookies as Bill. How many cookies does Laura have?

8. Jim walked 18 miles in 3 hours. How many miles did he walk per hour?

9. Maria spent $6 for 6 small soft drinks. How much did each drink cost?

10. Debbie walked 21 miles, which is 3 times as far as Jill walked. How far did Jill walk?

11. For his morning snack, John must choose one of 2 drinks offered and one of 3 foods offered. How many different snacks can he make? *Some children will find it easier to think of the choices as concrete things: milk or juice, pretzels, chips or grapes. Then they can write down the possibilities (milk with chips, milk with grapes, etc), if necessary.*

Embellished:
Be creative here!

12. *(larger numbers)* Each person should get 5 pencils. There are 430 pencils to give out. How many people will get pencils?

13. *(multistep)* Mary spent $15 at the concession stand. She bought 3 drinks at $2 each and some hamburgers. How much did she spend on the hamburgers?

14. *(extra information)* There are 15 girls, 12 boys, and 3 teachers on the bus. How many times more boys than teachers are there?